

Outline

- 14.1 Conduction of electricity
- 14.2 Drift velocity
- 14.3 <u>Current density</u>
- 14.4 Electric conductivity and resistivity

Objectives

g) derive and use the equation $\sigma = ne^2 t/m$

- h) define resistivity, and use the formula $\rho = RA/l$ i) show the equivalence between Ohm's law and
- the relationship J= σEj) explain the dependence of resistivity on temperature for metals and semiconductors
- by using the equation σ = ne²t/m
 k) discuss the effects of temperature change on the resistivity of conductors, semiconductors and superconductors.

Current and Charge Movement

Current-The rate at which electric charges move through a given area.

E. ELECTRICITY AND MAGNETISM

14. Electric current

Objectives

- a) define electric current, and use the equation I = dQ/dt
- b) explain the mechanism of conduction of electricity in metals
- c) explain the concept of drift velocity
- d)derive and use the equation I = Anev
- e) define electric current density and conductivity
- f) use the relationship $J = \sigma E$

14.1 Conduction of electricity

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Current and Charge Movement

- Whenever electric charges move, an electric current is said to exist
- The current is the rate at which the charge flows through a certain cross-section
- For the current definition, we look at the charges flowing perpendicularly to a surface of area *A*

Definition of the current:



Charge in motion through an area *A*. The *time rate* of the charge flow through *A* defines the current (=charges per time):

$I=\Delta Q/\Delta t$ Units: C/s=As/s=A

SI unit of the current: Ampere

Electric Current, cont

- The direction of current flow is the direction positive charge would flow
 - This is known as conventional (technical) current flow, i.e., from plus (+) to minus (-)
 - However, in a common conductor, such as copper, the current is due to the motion of the negatively charged electrons

Conventional Current

- The moving charges that make up current can be positive, negative, or a combination.
- In a conductor current is due to the motion of electrons.
- In the Physics Text, they use current as the movement of positive charges (earlier described by the instructor as hole theory).
- Ions can move positive charges, rather than individual electrons.

Type of Current

- Current can be Alternating or Direct
 - □ Alternating current (ac) continually changes direction.
 - Direct current (dc) charges move only in one direction.

Current

- SI unit for current is the *ampere*
- One ampere (amp, A) is equal to one coulomb of charge passing through an area in one second.
- $\blacksquare 1A = 1C/1s$

Electric Current, cont

 It is common to refer to a moving charge as a mobile *charge carrier* A charge carrier can be positive or negative 10

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Sources of Current

- Batteries and generators supply electrical energy to charge carriers.
 - □ Batteries convert *chemical* energy into electrical energy.
 - □ Generators convert *mechanical* energy into electrical energy

Electric Current



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nucleus

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electron

electron shell

> Electric circuits transport energy without moving any of its parts

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Electric Current

 \blacktriangleright Let us consider a battery of potential difference V, is connected across the ends of a conductor having length L

 \blacktriangleright An electric field **E** = **V**/**L** is established in the conductor



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Electric Current

> The electric current **i** is same for all cross-sections of a conductor, even though the cross-sectional area may be different at different points

> The direction of current is the direction that positive charges would move, even if the actual charge carriers are negative

> Even though we assign direction to current but is a scalar quantity, the arrow that we draw is just to show the sense of charge flow

found by integrate
$$\alpha = \int d d d$$

Electric Current

> The electric field is zero every where within a conductor

electron flo

electron

ucleus

copper aton

 \blacktriangleright In metals, like copper and aluminum some of electrons are free to move

> These free electrons move randomly in all directions

> There is no net flow of charge in any direction and hence no current

 \succ There is no force on electrons and no net flow of charges



Electric Current

> This electric field E acts on the electrons and gives them a net motion in the direction opposite to E

An electric current "i" has been established when a net charge dq passes through any surface in a time $i = \frac{dq}{dt}$ interval dt

➤ In SI units the unit of current is ampere "A"

> The net charge that passes through the surface in egrating the current any interval is

 $\mathbf{q} = \mathbf{j} \mathbf{i} \mathbf{d} \mathbf{t}$

Semiconductors

Properties of silicon

Property	copper	silicon	
Type of material	metal	semiconductor	
Charge carrier density	9×10 ²⁸	1×10 ¹⁶	
Resisitivity	2×10 ⁻⁸	3×10 ³	
Temperature coefficient of resistivity, K ⁻¹	+4×10 ⁻³	-70×10 ⁻³	



- Semiconductors are materials that exhibit a decrease in resistivity with an increase in temperature
- $\blacksquare \alpha$ is negative
- There is an increase in the density of charge carriers at higher temperatures



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Semiconductors

> In conductors, valence band is the highest band occupied by the electrons, which is partially filled and partially empty. There is a dividing line between the filled and empty state, the electrons in the filled states can easily jump (by applying some external electric field) to the upper empty states, which contribute to the current

> In insulator, the conduction band is empty and valence band is completely occupied by the electrons. The gap between the conduction band and valence band is very large i.e. about 2 ev or more than that. An electron may not be able to acquire such a large amount of energy from external source and jumps to the conduction band. Thus insulators are bad conductors



Semiconductors

Semiconductor

□ Pure semiconductor : high resistivity -> insulator □ Doping : two different impurities (n type, p type)

- Resistivity
 - In a conductor : increase of temperature -> increase in the collision rate -> decrease in -> increase of resistivity
 - □ In a semiconductor : increase of temperature -> increase in the collision rate , increase in the number of charge carriers -> decrease of resistivity *M*

 $\overline{e^2 n \tau}_{26}$



Semiconductors

➢ In semiconductor, the conduction band is completely empty and valence band is completely occupied. At ordinary temperature there is a small probability that an electron may jumps from the valence band to the conduction band by acquiring enough energy. This is possible because the energy gap between the conduction and valence of semiconductor is small, which is about 0.7 ev in case of germanium.

> The difference between the conductors and semiconductors is in the resistivity. Resistivity of the conductor increases with temperature while that of a semiconductor decreases with increase in temperature. The condctivity of the semiconductors changes by external factors such as temperature, applied voltage, incident light etc....

Drift Velocity

- Electrons do not move at the speed of light.
- When a switch is turned on charges from one end of the circuit push charges to the other end of the circuit.
- When a potential difference is placed on the conductor, and electric field is produced. (chp 17, 18)



Current density

- > The number of free electrons in the wire will be = **nAL**
- So charge will be q = (nAL)e, where e is the charge on single electron
- Thus much amount of charges passes through the wire in time t

 $t = L/V_d$

i = q/t

 \triangleright where current is

$$= \frac{\mathbf{nALe}}{\mathbf{L/v_d}} = \mathbf{nAev_d}$$

Current density

n = number of charges **e** per unit volume

i





The density of Cu atoms is $n = 8.49 \times 10^{28} m^{-3}$

Find v_d from $J = I/A = 1.3 A/(\pi (.0009m)^2) = 5.1 \times 10^6 A/m^2$

use
$$v_d = \frac{J}{ne} = \frac{5.1 \cdot 10^5 \, A/m^2}{(8.49 \cdot 10^{28} / m^3)(1.6 \cdot 10^{-19} C)}$$

 $v_d = 3.8 \times 10^{-5} \, m/s$

Much less than one millimeter per second!

Electrical conductivity

- The rate of charge flow (current) w/in a conductor depends on
 - a. The potential difference between 2 regions along the conducting pathway material (ΔV)
 - b. The cross sectional area of the conducting pathway (A)
 - c. The ability of the conductor to conduct charge (σ) {the conductivity}
 - d. The length of conducting pathway (L) where r is the "resistivity " (in units of or)

$$v_{d} = \frac{i}{nAe}$$
$$j = \frac{i}{A}$$
$$v_{d} = \frac{j}{ne}$$

Now adopting the convention for positive current density we can write the current density in the vector form as

$$\mathbf{j} = -\mathbf{ne} \mathbf{v}_{\mathbf{d}}$$

Example:

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- What is the drift velocity of electrons in a Cu wire 1.8 mm in diameter carrying a current of 1.3 A?
- In Cu there is about one conduction electron per atom. The density of Cu atoms is $n = 8.49 \times 10^{28} m^{-3}$

14.4 Electrical conductivity and Resistivity

Electrical conductivity For a uniform cylindrical conductor, combining these elements leads to the conduction equation: $\frac{dq}{dt} = i = \frac{\sigma A}{L} \Delta V = g \Delta V$ The quantity, $\frac{\sigma A}{L}$, is called the conductivity (g): $g = \frac{\sigma A}{L}$ Conductivity is more commonly expressed as the "resistance" (R) of the conducting pathway: $R = \frac{1}{g} = \frac{L}{\sigma A} = \frac{\rho L}{A}$ where r is the "resistivity" (in units of $\frac{V \cdot m}{A}$ or $\Omega \cdot m$)

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Superconductivity

Resistivity of the conductors is temperature dependent, increase in the temperature increases the resistivity of the conductors and vice versa. Resistivity of the conductor is due to the scattering of electrons by the vibrating atoms. Decrease in temperature decreases the vibration of atoms, which further deceases scattering of electrons with vibrating atoms, that results in decrease in the resistivity of the conductors.

- Superconductors
- A class of materials and compounds whose resistances fall to virtually zero below a certain temperature, T_C

 $\Box T_C$ is called the **critical temperature**

The graph is the same as a normal metal above T_C , but suddenly drops to zero at T_C

Superconductor Application

 $R(\Omega)$ 0.15

0.125

0.10

0.075

0.05

0.02

0.00

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4.0 4.1 4.2 4.3 4.4

Brooks Cold (K)

Hg

- An important application of superconductors is a superconducting magnet
- The magnitude of the magnetic field is about 10 times greater than a normal electromagnet

■Used in MRI units

Resistivity

The resistance of a wire is directly proportional to its length and inversely proportional to its cross-sectional area:

$$R = \rho \frac{L}{A}$$

The constant ρ , the resistivity, is characteristic of the material.

Superconductivity

If temperature of the conductors decreases to absolute zero, the vibrations of atoms decreases significantly and might be consider stationary, and the conductor will have zero resistivity. Such a conductors that have zero resistivity are called superconductor. It has been found experimentally that some materials behave superconductivity at 77K

50 49 Superconductors, cont The value of T_C is sensitive to: □chemical composition □pressure □molecular structure ■Once a current is set up in a superconductor, it persists without any applied voltage \Box Since R = o51 52 Superconductor ■ Superconductivity □ In 1911, Dutch physicist Kamerlingh Onnes □ The resistance of mercury □ In 1986, high temperature superconductor (new ceramic superconductor) Superconductor □ Normal conductors (ex, Silver, Copper) cannot become super conducting material at any temperature □ New ceramic super conductor Room temperature : good insulator Low temperature : superconductor 54 Resistivity TABLE 18-1 Resistivity and Temperature Coefficients (at 20°C) Resistivity, $\rho (\Omega \cdot m)$ Temperature Coefficient, α (C°)⁻¹ Material Conductors $1.59 imes 10^{-8}$ 0.0061 Silver Copper 1.68×10^{-8} 0.0068 Gold $2.44 imes 10^{-8}$ 0.0034 Aluminum 2.65×10^{-8} 0.00429 5.6 $\times 10^{-8}$ Tungsten 0.0045 $9.71 imes 10^{-8}$ 0.00651 Iron 0.003927 $10.6\ \times 10^{-8}$ Platinum 98×10^{-8} Mercury 0.0009

Values depend strongly on the presence of even slight amounts of impurities Copyright © 2005 Pearson Prentice Hall, Inc.

 100×10^{-8}

 $(3-60) \times 10^{-5}$

.1-60

 $10^9 - 10^{12}$

 $10^{13} - 10^{15}$

 $(1-500) \times 10^{-3}$

Nichrome (Ni, Fe, Cr alloy)

Semiconductors

Silicon

Insulators

Glass

55

Hard rubber

Carbon (graphite) Germanium 0.0004

-0.0005

-0.05

-0.07

Ohm's Law: Resistance and Resistors

Some clarifications:

- Batteries maintain a (nearly) constant potential difference; the current varies.
- Resistance is a property of a material or device.
- Current is not a vector but it does have a direction.
- Current and charge do not get used up. Whatever charge goes in one end of a circuit comes out the other end.

Resistivity

For any given material, the resistivity increases with temperature:

$$\rho_T = \rho_0 [1 + \alpha (T - T_0)]$$
 (18-4)

Semiconductors are complex materials, and may have resistivities that decrease with temperature.



- Semiconductors
- Superconductivity

Resistivity

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Resistivity

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Ohm's Law: Resistance and Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^{1}	
Red	2	10^{2}	
Orange	3	10^{3}	
Yellow	4	10^{4}	
Green	5	10^{5}	
Blue	6	10^{6}	
Violet	7	10^{7}	
Gray	8	10^{8}	
White	9	10^{9}	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
No color			20%

Resistance, Resistivity and Conductivity

The electrical resistance of a circuit component or device is defined as the ratio of the voltage applied to the electric current which flows through it

$$\mathbf{R} = \frac{\mathbf{V}}{\mathbf{i}} \tag{4}$$

We determine the resistance of a conductor between the two points by applying the potential difference V between those points and measure the current i that results.

If **V** is in volts and **i** in amperes, the resistance R is in volt/ampere which is called *ohms*

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Conductivity

> The electrons in a conducting materials are accelerated by the electric field E, Thus their drift velocity V_d is proportional to the electric field E

> Which means that the current density \mathbf{j} is proportional to the electric field \mathbf{E}

$$\vec{j} \propto \vec{E}$$
$$\vec{j} = \sigma \vec{E} \qquad (5)$$

> The proportionality constant σ is called the electrical conductivity of the material. It is not property of the particular sample of the material

Resistivity

Consider a conductor of length L having cross section area A , let ΔV is the potential applied across the two ends



There is a uniform electric field **E** and current density **j** produced in the conductor

$$E = \frac{\Delta v}{L}$$
 $j = -$

Ohm's law

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(7)

The current "i" passing through a conductor is directly proportional to the applied potential difference "V"

v = iR (8a)

If we plot current verses voltage for a conductor, the ratio V/i is always constant, which is called resistance.

Thus the resistance of a device is constant and is independent of the potential difference across it

ber in

Ohm's law

➤ If a conductor obeys Ohm's law its v versus i graph is linear and such conductors/devices are called Ohmic

➤ Its means that resistance /(resistivity) of material is independent of the applied voltage/ (Electric field)

 \succ If a conductor does not obey Ohm's law its v versus i graph is not linear and is called non-Ohmic like a pn junction diode

> The statement $\mathbf{v} = \mathbf{i}\mathbf{R}$ is not Ohm's law statement but it is a general definition of the resistance of a conductor whether it obeys Ohm's law or not

> The microscopic equivalent of the statement $\mathbf{v} = \mathbf{i}\mathbf{R}$ is

E = pj

Conductivity

> In SI units, the unit of conductivity is siemen/meter = S/m

> Where 1 Siemen = ampere/volt

Inverse of the conductivity is called resistivity, which is also the characteristic of materials

 $n = 1/\sigma$

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$$\vec{j} = \frac{\vec{E}}{\rho}$$
 (6)

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> The units of resistivity is ohm-meter i.e. Ω -m

> It must be noted that the resistivity is independent of the magnitude and direction of applied electric field

Resistivity

From equations 6 and 7 we have

$$\rho = \frac{\mathbf{E}}{\mathbf{j}}$$
$$\rho = \frac{\Delta \mathbf{V}/\mathbf{L}}{\mathbf{i}/\mathbf{A}}$$

Where $\Delta V/i = R$, Thus

$$= \rho \frac{\mathbf{L}}{\mathbf{A}} \tag{8}$$

The resistance of a conductor is independent of the magnitude and sign of the applied potential, it is the property of specific specimen of the sample.

Ohm's law

A conducting device obeys ohm's law if the resistance between any pair of points is independent of the magnitude and polarity of the applied potential difference



I.

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Microscopic view of Ohm's law

Ohm's law is not a fundamental electromagnetic law

In metal valence electrons are not attached to individual atoms but are free to move



These are called conducting electrons

Microscopic view of Ohm's law

> These freely moving electrons are some time called electron gas

> The electrical conduction in metals is based upon this freeelectron model

> Approximately the uniform average speed of electrons in the case of copper is 1.6 exp6 m/s

In ideal metallic crystal at 0 k electron-lattice collisions will not occur according to the prediction of quantum mechanics

 \succ Thus at T $\longrightarrow 0$ K,

 $> \lambda$ (Mean free path) $\longrightarrow \infty$

Microscopic view of Ohm's law

> When we apply electric field to a metal , the electrons modify their motion and they drift slowly in the opposite direction to that of the applied field

> This drift speed is very less as compared to the average speed of the electrons (about 10^{10})

 \blacktriangleright We can calculate the drift speed V_d in the terms of electric field E, average speed v and mean free path λ

Electric field E exert a force "eE" on electron, which imparts acceleration a to the electron

$$a = \frac{eE}{m}$$

Microscopic view of Ohm's law

$$V_{d} = \frac{\mathbf{j}}{\mathbf{ne}} = \frac{\mathbf{e} \mathbf{E} \boldsymbol{\tau}}{\mathbf{m}} \qquad (9)$$
$$\rho = \frac{\mathbf{E}}{\mathbf{i}} \qquad (10)$$

Now from equations 9, 10

$$\rho = \frac{\mathrm{m}}{\mathrm{n}\,\mathrm{e}^2\,\tau} \qquad (11)$$

In above equation all terms are constant except T, and it doesn't depend upon the applied electric field, so resistivity doesn't depend on E, which is the criteria for Ohm's law.

Its means that resistivity of material is independent of the applied field

 α is a parameter called temperature coefficient of resistivity.

 $R = R_{a} \left[1 + \alpha \left(T - T_{a} \right) \right]$



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Microscopic view of Ohm's law

Electron- Lattice collision in actual crystal

The ions are vibrating at any temperature about their equilibrium position

Impurities (foreign atoms) may be present

Their will be some missing atoms

So the resistivity of the metals may be increased due to

Raising its temperature

Adding small amounts of impurities

By increasing the number of lattice imperfections

Microscopic view of Ohm's law

The mean time between the two collisions is

$$\tau = \frac{\lambda}{\overline{x}}$$

Average change in the electron speed between two collisions is

$$a \frac{\lambda}{\pi}$$
 or $a\tau$

We define this to be the drift velocity of electron

$$V_d = a \tau = \frac{e E \tau}{m}$$

Where drift velocity in the terms of current density is

$$d = \frac{j}{ne}$$

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Temperature Variation of Resistance - Intro

- The resistivity of a metal depends on many (environmental) factors.
- The most important factor is the temperature.
- For most metals, the resistivity increases with increasing temperature.
- The increased resistivity arises because of larger friction caused by the more violent motion of the atoms of the metal.

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Temperature Variation of Resistance -Example

Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in the resistance of a conductor, is made of platinum and has a resistance of 50.0 Ω at 20°C. When the device is immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω . Find the melting point of Indium.

Solution:

Using $\alpha = 3.92 \times 10^{-3} (^{\circ}C)^{-1}$

