ELECTRICITY AND MAGNETISM

Chapter 16 Magnetic Fields

Outline

16.1 Concept of a magnetic field
16.2 Force on a moving charge
16.3 Force on a current-carrying conductor
16.4 Magnetic fields due to currents
16.5 Force between two current-carrying conductors
16.7 Determination of ratio q/m
16.8 Hall effect

Objectives

a) explain magnetic field as a field of force produced by current-carrying conductors or by permanent magnets
b) use the formula for the force on a moving charge, \( F = qv \times B \)
c) use the equation \( F = qvB \sin \theta \) to define magnetic flux density \( B \)
d) describe the motion of a charged particle parallel and perpendicular to a uniform magnetic field
e) explain the existence of magnetic force on a straight current-carrying conductor placed in a uniform magnetic field
f) derive and use the equation \( F = IlB \sin \theta \)

Objectives

• explain the principles of the determination of the ratio \( e/m \) for electrons in Thomson’s experiment (quantitative treatment is required)
• understand the principles of determination of the ratio for charged particles \( q/m \)
• explain the Hall effect and derive an expression for Hall Voltage \( V_H \)
• state the applications of Hall effect

Magnetic field \( B \)

- A **magnetic field** exists in the space around a magnet
  - Another magnet placed near it experiences a **force**
- A magnet always has 2 poles, labeled **north** (N) and **south** (S)
  - No one has yet found a **magnetic monopole** (an isolated N or S pole)
- Like poles repel, and unlike poles attract

Lodestones

• The first known (permanent) magnets were the naturally occurring **lodestones**, pieces of iron mineral known today as **magnetite**
Ferromagnetic Materials
- A (permanent) magnet retains its magnetic properties for a long time.
- Substances made of iron, nickel, cobalt, or mixtures of these elements, are said to be ferromagnetic.
- When placed near a magnet, an unmagnetized ferromagnetic material becomes magnetized and can induce magnetism in other unmagnetized ferromagnetic materials nearby.
- If subjected to a very strong magnetic field, a ferromagnetic object can become a permanent magnet.

Magnetic Field
- A magnetic field has both magnitude and direction.
- The field direction at any point in space is the direction indicated by the north pole of a small compass needle placed at that point.

Magnetic Field Examples

Magnetic Field Lines
- Imaginary magnetic field lines provide information on the direction and strength of the field at any point.
- The field lines appear to originate from the north pole and end on the south pole.

Magnetic Field Units
We can determine the magnetic field by measuring the force on a moving charge:

\[ B \equiv \frac{F}{qv \sin \theta} \]

The SI unit: Tesla (T)
OR Gauss (G) where \( 1 \text{T} = 10^4 \text{G} \)
The earth’s magnetic field is about \( 0.5 \text{ G} \)
Dimensional analysis:
\( 1 \text{T} = 1 \text{N} \cdot \text{s} / (\text{C} \cdot \text{m}) = 1 \text{V} \cdot \text{s} / \text{m}^2 \)

How in the world am I going to remember whether I should use dots or \( x \)'s?

The tail of an arrow.

The tip of an arrow.

The \( x \)'s

The dots
Earth’s Magnetic Field
- The earth behaves magnetically almost as if a bar magnet were located near its center.
- The south pole of the fictitious bar magnet is in the northern hemisphere.

Force on Moving Charge in Magnetic Field
- Experiments show that a charge $q$ placed in a magnetic field experiences a force if:
  - $q$ is moving, (no magnetic force acts on a stationary charge) and
  - the velocity of $q$ has a component perpendicular to the direction of the field.

Magnetic Force
- If a charge $q_o$ moves parallel or antiparallel to the field, $q_o$ experiences no magnetic force.
- If $q_o$ moves perpendicular to the field, $q_o$ experiences the maximum possible force $F_{\text{max}}$.
- In general, if $q_o$ moves at an angle $\theta$ with respect to the field, only the velocity component $v \sin \theta$ gives rise to a magnetic force.

Right-Hand Rule #1
- The direction of the magnetic force $F$ is always perpendicular to both the velocity $v$ and the magnetic field $B$.
- The directions of $F$, $v$, and $B$ for a positive charge follow the Right-Hand Rule No. 1 (RHR-1), as illustrated in the figure.
- For a negative charge, the direction of $F$ is reversed.

Magnitudes of Magnetic Force & Field
- The magnitudes $F$ & $B$ of the magnetic force & field are related to the speed $v$ of the charge $q_o$ by $F = q_o v B \sin \theta$.
- The SI unit for magnetic field is tesla (T), where 1 T = 1 N-s/(C-m).

Motion of Charged Particle in Magnetic Field
- The electric force on a charge
  - is parallel or antiparallel to the electric field
  - generally changes the speed & possibly also the direction of the motion.
- The magnetic force on a moving charge is always perpendicular to its velocity
  - causing its direction to change
  - leaving its speed unchanged.
16.3 FORCE ON A CURRENT-CARRYING CONDUCTOR

**Force on a Wire**
- The blue x's indicate the magnetic field is directed into the page
  - The x represents the tail of the arrow
- Blue dots would be used to represent the field directed out of the page
  - The • represents the head of the arrow
- In this case, there is no current, so there is no force

**Force on a Wire**
- B is into the page
- The current is up the page
- The force is to the left

**Force on a Wire**
- B is into the page
- The current is down the page
- The force is to the right

**Force on a Wire, equation**
- The magnetic force is exerted on each moving charge in the wire
- The total force is the sum of all the magnetic forces on all the individual charges producing the current
- \( F = B I \ell \sin \theta \)
  - \( \theta \) is the angle between \( B \) and the direction of \( I \)
  - The direction is found by the right hand rule, placing your fingers in the direction of \( I \) instead of \( v \)

**Magnetic Force on a Power Line**
- A force is exerted on a current-carrying wire placed in a magnetic field
  - The current is a collection of many charged particles in motion
- The direction of the force is given by right hand rule #1
16.4 MAGNETIC FIELDS DUE TO CURRENTS

**Definition of the Ampere**
- The force between two parallel wires can be used to define the ampere.
- When the magnitude of the force per unit length between two long parallel wires that carry identical currents and are separated by 1 m is \(2 \times 10^{-7} \text{ N/m}\), the current in each wire is defined to be 1 A.

**Ampère’s Law**
- André-Marie Ampère found a procedure for deriving the relationship between the current in an arbitrarily shaped wire and the magnetic field produced by the wire.
- Ampère’s Circuitual Law:
  - \(\sum B_{||} \Delta \ell = \mu_0 I\)
  - Sum over the closed path

**Ampère’s Law to Find B for a Long Straight Wire**
- Use a closed circular path.
- The circumference of the circle is \(2\pi r\).
  \[B = \frac{\mu_0 I}{2\pi r}\]
- This is identical to the result previously obtained.
- General form of Ampere’s law:
  \[\oint B \cdot dl = \mu_0 I_{\text{enc}}\]
  ⇒ Field inside & outside a wire, coaxial cable

**Definition of the Coulomb**
- The SI unit of charge, the **coulomb**, is defined in terms of the ampere.
- When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.

**Magnetic Field Produced by Currents**
- In 1820, Oersted first discovered that a wire carrying a current also produces a **magnetic field of its own**.
- This marked the start of a very important discipline now called **electromagnetism**.

**André-Marie Ampère**
- 1775 – 1836
- Credited with the discovery of electromagnetism:
  - Relationship between electric currents and magnetic fields
- Mathematical genius evident by age 12

**Ampère’s Law, cont**
- Choose an arbitrary closed path around the current.
- Sum all the products of \(B_{||} \Delta \ell\) around the closed path.

**Straight Wire**
- \(I\) is the current.
- The field is given by
  \[B = \frac{\mu_0 I}{2\pi r}\]
- This is identical to the result previously obtained.
- General form of Ampere’s law:
  \[\oint B \cdot dl = \mu_0 I_{\text{enc}}\]
  ⇒ Field inside & outside a wire, coaxial cable
Magnetic Field Produced by Long, Straight Wire
- Experiments show that the magnitude $B$ of the magnetic field produced by a long, straight wire carrying a current $I$ is directly proportional to $I$ and inversely proportional to the radial distance $r$ from the wire: $B \propto \frac{I}{r}$

- In equation form $B = \frac{\mu_0 I}{2\pi r}$

  where $\mu_0 = 4\pi \times 10^{-7}$ T·m/A is known as the permeability of free space

Right-Hand Rule #2
- The direction of the magnetic field $B$ produced by a long, straight wire carrying a current $I$ follows the Right-Hand Rule No. 2 (RHR-2), as illustrated in this figure

Solenoids
- If we stack several current loops together we end up with a solenoid:

  In the limit of a very long solenoid, the magnetic field inside is very uniform:

  $$B = \mu_0 n I$$

  $n$ = number of windings per unit length, $I$ = current in windings

  $B \approx 0$ outside windings

Magnetic Field due to a Solenoid
- The magnetic field is strongest at the centre of the solenoid and becomes weaker outside.

  $B = \mu_0 n I$

  $B \approx 0$ outside windings

Magnetic Flux Density due to a Solenoid
- Experiments show that the magnetic flux density inside a solenoid is

  $B \propto I$ and $B \propto \frac{N}{l}$

  So we have $B = \frac{\mu_0 n I}{l}$

  or $B = \mu_0 n I$

  where $n = \frac{N}{l}$

Variation of magnetic flux density along the axis of a solenoid
- $B$ is independent of the shape or area of the cross-section of the solenoid.
- At a point at the end of the solenoid,

  $B = \mu_0 n I$

  $B' = \frac{1}{2} \mu_0 n I$

  Distance from the centre of the solenoid

16.5 FORCE BETWEEN TWO CURRENT-CARRYING CONDUCTORS
Magnetic Force Between Two Parallel Conductors, final

- The result is often expressed as the magnetic force between the two wires, $F_B$
- This can also be given as the force per unit length, $F_B/l$

\[
F_B = \frac{\mu_0 I_1 I_2}{2\pi a}
\]

Magnetic Force Between Two Parallel Conductors, Cont.

- The force per unit length is then:

\[
\frac{F_1}{l_1} = B_2 I_1 = \frac{\mu_0 I_1 I_2}{2\pi d}
\]

$\Rightarrow$ Definition of the Ampere:
If two long, parallel wires 1m apart carry the same current, and the magnetic force per unit length on each wire is $2 \times 10^{-7}$ N/m, then the current is defined to be 1 A.

Quick Quiz 19.5

- If the currents $I_1 = 2$A and $I_2 = 6$ A, which of the following is true?
  - (a) $F_1 = 3 F_2$
  - (b) $F_1 = F_2$
  - (c) $F_1 = F_2/3$

  Answer (b), since $F \sim I_1^2$.

16.6 DETERMINATION OF RATIO $e/m$

- What produces a gravitational field? Mass
- A gravitational field exerts a force on? Mass
- What produces an electric field? Electric charge
- An electric field exerts a force on? Electric charge
- What produces a magnetic field? Moving electric charge
- A magnetic field exerts a force on?
- Moving electric charge?

It was found that the cathode rays could be deflected by an electric or magnetic field. The direction of the deflection was consistent with a negative charge.
Mass Spectrometer
- Application of equation for trajectory of charged particle in a constant magnetic field
- Magnetic Force on a current-carrying wire
- Current Loops
  - Magnetic Dipole Moment
  - Torque (when in constant B field) → Motors
  - Potential Energy (when in constant B field)
- Nuclear Magnetic Resonance Imaging

Others application
- Thomson (1897) measures \( q/m \) ratio for “cathode rays”
  - All have same \( q/m \) ratio, for any material source
  - Electrons are a fundamental constituent of all matter!
- Accelerators for particle physics
  - One can easily show that the time to make an orbit does not depend on the size of the orbit, or the velocity of the particle
  - *Cyclotron*

Mass Spectrometer, cont.
- Electrically detect the ions which “made it through”
- Change B (or V) and try again:

Mass Spectrometer
- Moving charged particles are deflected in magnetic fields \( \vec{F} = q \vec{v} \times \vec{B} \)
- Circular orbits \( R = \frac{mv}{qB} \)
- If we use a known voltage \( V \) to accelerate a particle
  \[ qV = \frac{1}{2}mv^2 \rightarrow \frac{q}{m} = \frac{2V}{R^2B^2} \]

Mass Spectrometer
- Measure \( m/q \) to identify substances
- Electrostatically accelerated electrons knock electron(s) off the atom → positive ion \((q=|e|))
- Accelerate the ion in a known potential \( U = qV \)
- Pass the ions through a known B field
  - Deflection depends on mass: Lighter deflects more, heavier less

Another example
- Measuring curvature of charged particle in magnetic field is usual method for determining momentum of particle in modern experiments:
  - e.g.

Direction of Magnetic Force
Direction of magnetic force is “sideways”
- force is perpendicular to both \( v \) and \( B \)
- use “right-hand rule” to find direction

Radius of Circular Orbit
- magnetic force: \( F = qvB \)
- centripetal accel \( a = \frac{v^2}{R} \)
- Newton’s 2nd Law:
  \[ F = ma \]
  \[ qvB = m\frac{v^2}{R} \]
  \[ R = \frac{mv}{qB} \]
- This has useful experimental consequences!
αs are ionized Helium (bare Helium nuclei)
2-protons, 2-neutrons (positively charged)
βs are simply electrons (negatively charged)
\[ q_\alpha = -2q_\beta \]
\[ m_\alpha = 7296m_\beta \]

**Velocity Selector**

Consider a positively charged ion entering a region where the electric and magnetic fields are uniform and perpendicular to each other. If the particle moves in a straight line, what is its velocity in terms of E and B?

\[ v = \frac{E}{B} \]

**Ratio of charge to mass for an electron**

An electron is accelerated from rest across a potential difference and then enters a region of uniform magnetic field, as shown at right. What is the “charge to mass ratio”, q/m, of the electron?

\[ F_E = qE \]
\[ F_B = qvB \]

\[ qvB = qV/d \]

What is the radius of the electron’s orbit?

\[ R = \frac{mv}{qB} \]

**What is the hall effect?**

Just the basics:

The change in magnetic field induces a current, the change in intensity and direction of the current can measure the velocity and direction object producing the magnetic field.

The basic physical principle underlying the Hall effect is the Lorentz force.
**Hall effect in details**

When an electron moves along a direction perpendicular to an applied magnetic field, it experiences a force acting normal to both directions and moves in response to this force and the force effected by the internal electric field.

**Hall Voltage**

- The transverse voltage builds up until the electric field it produces exerts an electric force on the moving charges that equal and opposite to the magnetic force.
- The transverse voltage produced is called the Hall voltage.

**Hall Probe**

- Basically the Hall probe is a small piece of semiconductor layer.
- Four leads are connected to the midpoints of opposite sides.
- When control current IC is flowing through the semiconductor and magnetic field B is applied, the resultant Hall voltage VH can be measured on the sides of the layer.

**Hall Effect**

- When a current carrying conductor is held firmly in a magnetic field, the field exerts a sideways force on the charges moving in the conductor.
- A buildup of charge at the sides of the conductor produces a measurable voltage between the two sides of the conductor.
- The presence of this measurable transverse voltage is called the Hall effect.

**Charge Carriers in the Hall Effect**

- The Hall voltage has a different polarity for positive and negative charge carriers.
- That is, the Hall voltage can reveal the sign of the charge carriers.

**The Hall Effect**

When a charged particle moving in a vacuum, it is deflected perpendicular to its velocity by a magnetic field. In 1879, Edwin Hall, a graduate student at Johns Hopkins Univ., discovered that the same behavior is true of charged particles moving in a conductor.

**Hall Effect**

- There is another effect that occurs when a wire carrying a current is immersed in a magnetic field.
- Assume that it is the positive charges that are in motion
- These positive charges will experience a force that will cause them to also move in the direction of the force towards the edge of the conductor, leaving an apparent negative charge at the opposite edge.

**Hall Effect**

- The fact that there is an apparent charge separation produces an electric field across the conductor.
- Eventually the electric field will be strong enough so that subsequent charges feel an equivalent force in the opposite direction.
- Since there is an electric field, there is a potential difference across the conductor which is given by

\[ V = E_e d = v_d B d \]
Hall Effect

- The Hall Effect allows us to determine the sign of the charges that actually make up the current
- If the positive charges in fact constitute the current, then potential will be higher at the upper edge
- If the negative charges in fact constitute the current, then potential will be higher at the lower edge
- Experiment shows that the second case is true
- The charge carriers are in fact the negative electrons

Hall Effect

- From the equation $V = E_v d = v_d B d$
  - The drift speed, $v$ can be measured
- From the equation $V = \frac{BI}{nea}$
  - The number of charge carries per unit volume, $n$ can be measured
  - The magnetic field strength $B$ can be measured

Summary: Magnetic Field

<table>
<thead>
<tr>
<th>Force</th>
<th>Magnetic Field</th>
<th>Determination of $e/m$</th>
<th>Hall Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A moving charge: $F = q(v \times B)$</td>
<td>• Straight wire, $B = \frac{\mu I}{2\pi d}$</td>
<td>• $e/m = \frac{2V}{B^2R^2}$</td>
<td>• Hall voltage, $V_H = \frac{IB}{ned}$</td>
</tr>
<tr>
<td>• CCC (Current-Carrying Conductor): $F = I(I \times B)$</td>
<td>• Circular loop, $B = \frac{\mu NI}{2r}$</td>
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</tr>
<tr>
<td>• Between 2 CCC: $F = \frac{I_1 I_2}{d}$</td>
<td>• Stolenoid, $B = \frac{\mu nI}{l}$</td>
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